An Introduction to the Nikkei 225 Implied volatility index

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ABSTRACT

This paper introduces some benchmarks of volatility implicit in Nikkei 225 stock option prices that are not available on financial and economic databases. The construction of implied volatility indices for the Japanese stock market is based on the different the methodologies followed in calculating the original and new VIX for the S&P 100 and S&P 500 indices by the Chicago Board of Options Exchange. The time-series reveal patterns of divergence in the early 1990s and recent convergence of implied volatility indices across countries. As far as the Nikkei 225 implied volatility is concerned, the empirical evidence suggests that the level of implied volatility is sensitive to past observations of returns and volatility expectations in the US market. Implied volatility is also likely to increase in bearish markets and decrease in bullish markets. The results also indicate that the dynamics of implied volatility in both markets are governed by mean reversion, which suggests that markets have a memory for past levels of implied volatility.

1. Introduction

Risk management is an important issue not only from the perspectives of investment portfolio managers but for the purposes of market regulation and

* Acknowledgement: This paper is dedicated to Professor Toshihiro Kojima whose useful advice and continuous support are highly appreciated and hereby thankfully acknowledged.

monetary policy as well. In addressing some crucial questions related to risk management, there are complicating factors which derive from the complexity of risk-hedging instruments, numerical analysis and the dynamic linkage between international financial markets. Derivative instruments including currency and stock index options can be useful in providing inferences about future market volatility. However, volatility expectations tend to diverge as option premia differ depending on exercise prices and options expirations. These discrepancies in implicit volatility are reflected in volatility smiles and sneers, which are inconsistent with the theoretical modeling of options on the assumption of constant volatility. It is possible to construct an index of implied volatility in order to provide an aggregate estimate of implied volatility, representative of option market expectations. Such benchmarks of implied volatility can be developed following two distinct approaches, based either on a particular theoretical model of option pricing or on a model-free methodology. Using either benchmark, some important empirical issues can be examined. It is possible for instance to shed light on the dynamics of volatility expectations, the patterns in which it flows and ebbs, and how it differs across international markets.

The early evidence from Day and Lewis (1992) indicates that implied volatility from S&P 100 index options contains some incremental information, which is useful for inference purposes. Using volatility estimates implicit in individual stock options, Lamoureux and Lastrapes (1993) also provide evidence in support of the incremental information about future volatility contained in option prices. The empirical results by Fleming (1998) are also supportive of the significance of implied volatility for forecasting purposes, despite evidence of upward bias. Based on out-of-sample forecasting, there is further evidence from Blair, Poon and Taylor (2001) that implied volatility S&P 100 VIX index is more reliable than realized volatility. An important exception in this growing literature on the merits of implied volatility is the evidence from Canina and Figlewski (1993), who reject the informational usefulness of S&P 100 implied volatility, finding it neither informative on realized volatility nor accurate for forecasting purposes.

The centricity of this body of evidence on US markets does not allow for the drawing of general conclusions valid for other international stock markets. To provide a broader perspective on volatility expectations, this study introduces newly constructed implied volatility indices for the Japanese stock market. The volatility indices are derived from Nikkei 225 index options traded on Osaka Securities Exchange following the methodology used in calculating the S&P 100 and S&P 500 implied volatility indices by the Chicago Board of Options Exchange. The Nikkei 225 stock index is a major benchmark for the Japanese stock market, and constitutes the underlying asset of various financial derivatives trading on three Asia-Pacific derivatives markets, namely Osaka Securities Exchange, Chicago Mercantile Exchange and Singapore Exchange.

Using these indices of implied volatility, this study examines the distributional properties of volatility expectations from options markets. It also analyses the dynamics of implied volatility in light of its relationship with stock market returns. The analysis of stochastic properties is aimed at identifying patterns in which the formation of volatility expectations differs across international markets. The remainder of the paper is organized as follows. The next section reviews the different methodologies underlying the construction of these implied volatility indices. The distributional properties of implied volatility are discussed in section 3. Section 4 examines the dynamics of implied volatility in its relationship with stock market returns. Section 5 concludes the paper.

2. Methodological Issues in Constructing Implied Volatility Indices

The objective of this section is to describe the various methodologies underlying the computation of implied volatility benchmarks for the Japanese stock market. The first part introduces the approach for computing the original index of implied volatility from S&P 100 options using Black-Scholes option pricing model while the second part focuses on the model-free approach underlying the calculation of S&P 500 new volatility index. The implied volatility indices for the Japanese market are computed using the daily closing prices of the Nikkei 225 call and put options. The OSE-traded Nikkei 225 options expire on the second Friday of the contract month. Starting form the inception of options trading in June 12, 1989, the sample period extends until June 8, 2001.

2.1. The Original VIX Implied Volatility Index

The original implied volatility index is a measure of expected volatility implicit in the price of a theoretical option with exercise price equal to the underlying asset and with 30 calendar days remaining to expiration. Its calculation is based on at-the-money call and put options spanning the nearest maturities and as such, it takes into account the term structure of implied volatility as well as the volatility smiles and sneers, which reflect differences in implied volatility estimates depending on exercise prices. The methodology appears as a variant of weighting schemes applied to estimates of implied volatility. Volatility is estimated from option prices through numerical analysis following Latane and Rendleman (1976) based on the Black-Scholes option pricing model and the dividend-augmented version by Merton (1973). The competing measures of daily implied volatility depend on exercise prices and time remaining to expiration. In order to arrive at a single daily measure of implied volatility, these various estimates are weighted on the basis of elasticity, simple averaging or vega parameter, the partial derivative of option premia with respect to volatility. The approach followed by CBOE in determining the S&P 100 implied volatility is closer to the vega-weighting scheme in the sense that it relies upon the most liquid near-term and next-term options, which are also associated with higher vega estimates.

The methodology underlying the S&P 100 implied volatility index is outlined in Whaley (2000). The selection of near-term and next-term options on a given trading date is subject to the rollover process which is triggered when the time remaining to maturity falls below the critical limit of eight calendar days. This rollover process is meant to avoid measurement problems deriving from short-lived options which are usually associated with extreme estimates of implied volatility. Thus, the application of this rollover rule practically eliminates options with less than 8 calendar days, or approximately 6 trading days, from the sample. The focus is also made on near-the-money options which are associated with higher time premium than deep-in-the-money or deep-out-of-the-money options and are, at least theoretically, more sensitive to changes in volatility. The estimation of implied volatility from each option price is based on time to expiration being expressed in calendar days. This estimate is subsequently converted into volatility based on the number of trading days with the approximation formula $v_t = v_t\sqrt{\tau_c / \tau}$ where $\tau = \tau_c - 2\mathrm{int}(\tau_c / 7)$ represents the approximate number of trading days.

The estimation process starts with the identification of the upper exercise price K_u and lower exercise price K_ℓ immediately surrounding the observed stock index price. For the near-term expiration m_1, the corresponding call and put premia for options with upper exercise price K_u are denoted as $C_u^{m_1}$ and $P_u^{m_1}$, respectively. Likewise, the corresponding call and put premia for options with lower exercise price K_l are denoted as $C_l^{m_1}$ and $P_l^{m_1}$, respectively. For next-term expiration m_2, the call and put premia for options with upper exercise price K_u are denoted as $C_u^{m_2}$ and $P_u^{m_2}$, respectively while $C_l^{m_2}$ and $P_l^{m_2}$ denote the respective options with lower exercise price K_l. This identification process results in eight measures of implied volatility that differ across option types, maturities and exercise prices.

The volatility implicit in these option prices is derived by numerical methods based on Merton's dividend-adjusted version of Black-Scholes option pricing formula

$$C = Se^{-\delta\tau}N(d_1) - Ke^{-r\tau}N(d_2)$$

$$P = Ke^{-r\tau}N(-d_2) - Se^{-\delta\tau}N(-d_1) \qquad (1)$$

where S is the current stock price, δ is the expected dividend yield, which is arbitrarily estimated using the annual average yield of 0.5% on all stocks listed on Tokyo Stock Exchange. The exercise price is denoted as K while τ is the time remaining to expiration. $N(.)$ is the standard normal distribution function, $d_1 = \dfrac{ln(S/K)[(r+\sigma^2/2)\times\tau]}{\sigma\sqrt{\tau}}$ whereas $d_2 = d_1 - \sigma\sqrt{\tau}$. The continuous yield on the three-month or one-month Certificate of Deposit associated with the closest maturity to the expiration date is used as proxy for the interest rate r.

The estimates of volatility implicit in call options $v_t^c(m_i, K_j)$ and put options $v_t^p(m_i, K_j)$ with different exercise prices and expirations are subject to successive averaging and interpolation processes in order to obtain a single value of the implied volatility index. This averaging takes place to reduce differences at three different dimensions, with respect to option type, strike price and time to expiration. The first averaging takes into account differences across options types in terms of call and put options

$$v_t(m_i, K_j) = \frac{v_t^c(m_i, K_j) + v_t^p(m_i, K_j)}{2} \qquad (2)$$

The following step is to interpolate the two pairs of implied volatilities based on the nearest-maturity $v_t(m_1, K_j)$ and second-nearest-maturity options $v_t(m_2, K_j)$. Applied for each maturity month, this interpolation process represents an adjustment process that smoothes out differences between exercise prices and the observed stock index level. It results in the approximation of implied volatility to the hypothetical at-the-money option with exercise price equal to the underlying index.

$$v_t(m_i) = v_t(m_i, K_u)\frac{K_u - S_t}{K_u - K_l} + v_t(m_i, K_l)\frac{S_t - K_l}{K_u - K_l} \quad (3)$$

The final step is to extrapolate implied volatility estimates across different maturities in order to approximate its value for a hypothetical option of exactly 30 calendar days, or 22 trading days to expiration. The adjustment process takes place with respect to the implied volatility estimates associated with near-term option with τ_1 trading days remaining to expiration and next-term options with τ_2 trading days to expiration, according to the following equation.

$$v_t = v_t(m_1)\frac{\tau_2 - 22}{\tau_2 - \tau_1} + v_t(m_2)\frac{22 - \tau_1}{\tau_2 - \tau_1} \quad (4)$$

The implementation of this methodology with respect to the Nikkei 225 index options poses some difficulties, which add to the potential misspecification of Black-Scholes option pricing model and problems of non-synchronous trading across the stock exchange and options market. The initial period of OSE option trading, during the late 1980s in particular, is associated with problems of non-convergence in the numerical procedure which inverts volatility from closing option prices. These complications may be related to option mispricing and/or measurement problems, and have the effect of hindering the averaging and interpolation process.

For instance, the methodology is based on the premise that the lower and upper exercise prices, which are function of option type, do not differ across call and put options with the same maturity, $K_j(C) = K_j(P)$, which is not always the case. It is not clear how to proceed in the event that no put option can be identified to match either the upper or lower exercise price associated with call options. In such cases, it is not possible to strictly abide by the methodological procedure without compromising the averaging process in the presence of differences in exercise prices across option types. When the problems of numerical convergence are encountered with respect to closing prices, the recourse is made to implied volatility from opening, highest or lowest option prices in order to minimize the loss of information from option trades. The opening and high option prices are used in 5 cases for call options and 4 cases for put options.

Because of data limitations in early options trades, there are also problems in implementing the interpolation process when either the implied volatility from near-term or next-term options is equal to zero $v_t(m_i) = 0$. In these cases, the approximation is made under the assumption that volatility is function of the square root of time. The level of implied volatility index is approximated as $v_t = v_t(m_2)\sqrt{\tau_2/22}$ when $v_t(m_1) = 0$ and vice-versa.

2.1. The New VIX Implied Volatility Index

This study develops another index of volatility expectations in the Japanese stock market following the methodology used for computing the new VIX index for S&P 500 options by the Chicago Board of Options Exchange. The new benchmark of volatility expectations is based on a hypothetical option of 30 days to expiration and is computed using a range of exercise prices for call and put options spanning the nearest and next maturities. For the purposes of comparison, the methodology based on online CBOE publications is reviewed below. The explanation of the various methodological steps is accompanied with an illustrative example based on the closing trades of Nikkei 225 index options on February 27, 2001.

As with the methodology underlying the S&P 100 implied volatility index, the approach for calculating the new implied volatility index for S&P 500 index is based on the selection of options with near-term and next-term maturities. Again, the rollover process to the next maturity is triggered when the time remaining to expiration falls below 8 calendar days. In the example of February 27, 2001 options, the near-term options have 11 calendar days to expiration while next-term options are associated with 46 remaining days. Thus, the process of computing the implied volatility index can take place without the rollover to the second and third contract maturities.

The first step in the calculation methodology is to determine the at-the-money exercise price, which is associated with the minimal difference between call and put option prices. In this example, at-the-money exercise prices coincide with $K^{\bullet}$ equal to 13000 yen, which is associated with the minimum spread between call and put option premia of 25 and 45 for the near and next term maturities, respectively. As shown in Table 1, this minimum spread is determined on the basis of the absolute value of call-put price differences. The procedure allows for at-the-money exercise prices to differ across maturities.

The at-the-money strike price is used to estimate the Nikkei 225 forward

Table 1. Determination of at-the-money exercise prices for near- and next-term maturities

Near-Term Options March 2001 (11 Days to Expiration)				*Next-Term Options April 2001 (46 Days to Expiration)*			
Exercise Price	Call Premium	Put Premium	Call-Put Spread	Exercise Price	Call Premium	Put Premium	Call-Put Spread
11000	2100	-	-	11000	2140	15	2125
11500	1700	3	1697	11500	1660	35	1625
12000	1060	10	1050	12000	1150	90	1060
12500	560	45	515	12500	745	195	550
$K^{\bullet}=13000$	205	180	25	$K^{\bullet}=13000$	380	335	45
13500	40	500	-460	13500	170	550	-380
14000	10	930	-920	14000	75	900	-825
14500	3	1300	-1247	14500	30	1380	-1350
15000	2	1730	-1728	15000	15	1870	-1855
15500	-	2350	-	15500	6	2370	-2364
16000	-	2850	-	16000	-	2840	-
16500	-	3330	-	16500	-	3360	-
17000	-	3820	-	17000	-	3850	-
17500	-	4370	-	17500	-	-	-

price as $F = K^{\bullet} + e^{r\tau}(C - P)$. It is important to note that the cost-of-carry formula to define forward prices is not based on spot prices but on the exercise prices associated with the minimum difference between call and put options. The time-to-expiration τ is equal to the total number of minutes from the trading date to the settlement date and intervening days, scaled by the number of minutes per year. The forward levels are determined for both the near-term and next-term maturities as $F_1 = 13025.68$ and $F_2 = 13050.87$, respectively. This allows for the determination of the exercise price K_0 that immediately precedes the forward price. This strike price K_0 coincides with $K^{\bullet} = 13000$ for both the near-term and next-term maturities.

Table 2. Determination of option premium for near- and next-term maturities

NEAR-TERM EXERCISE PRICE	OPTION TYPE	OPTION PRICE	NEXT-TERM EXERCISE PRICE	OPTION TYPE	OPTION PRICE
-	-	-	11000	PUT	15
11500	PUT	3	11500	PUT	35
12000	PUT	10	12000	PUT	90
12500	PUT	45	12500	PUT	195
13000	PUT&CALL	192	13000	PUT&CALL	357
13500	CALL	40	13500	CALL	170
14000	CALL	10	14000	CALL	75
14500	CALL	3	14500	CALL	30
15000	CALL	2	15000	CALL	15
-	-	-	15500	CALL	6

The following step is to select only in-the-money call options with $K > K_0$ and in-the-money put options with $K < K_0$. Upon ranking all options in increasing order of exercise price, it is the put option prices that are taken into consideration followed by call option premia. As indicated in Table 2, the option premium for $K = K_0$ is equal to the average of call and put premia, amounting to $192 = (205+180)/2$ and $357 = (380+335)/2$ for near-term and next-term maturities, respectively. It is noted that the range of exercise prices and the number of options are allowed to differ across maturities depending on the availability of simultaneous price observations of call and put options needed to calculate option price differences.

Table 3A. Option contribution to implied volatility for near-term implied volatility

Near-Term Options March 2001 (11 Days to Expiration)				
Near-Term Exercise Price	Option Type	Option Price	Individual Option Contribution	Cumulative Option Contribution
11500	PUT	3	0.000011	0.000011
12000	PUT	10	0.000035	0.000046
12500	PUT	45	0.000144	0.000190
13000	PUT&CALL	192	0.000568	0.000758
13500	CALL	60	0.000110	0.000868
14000	CALL	10	0.000026	0.000894
14500	CALL	3	0.000007	0.000901
15000	CALL	2	0.000004	0.000905

Table 3B. Option contribution to implied volatility for next-term implied volatility

Next-Term Options April 2001 (46 Days to Expiration)				
Near-Term Exercise Price	Option Type	Option Price	Individual Option Contribution	Cumulative Option Contribution
11000	PUT	15	0.000062	0.000062
11500	PUT	35	0.000132	0.000194
12000	PUT	90	0.000313	0.000507
12500	PUT	195	0.000624	0.001132
13000	PUT&CALL	357	0.001057	0.002188
13500	CALL	170	0.000467	0.002655
14000	CALL	75	0.000191	0.002847
14500	CALL	30	0.000071	0.002918
15000	CALL	15	0.000033	0.002951
15500	CALL	6	0.000012	0.002964

Using this structure of option prices $Q(K_n)$ which is function of exercise prices, the contribution of the n^{th} option to the new implied volatility index is expressed as $(\triangle K_n/K_n^2)e^{r\tau_m}Q(K_n)$. The spread between exercise prices $\triangle K_n$ represents the average of the surrounding K_{n-1} and K_{n+1} strikes prices. At the lower and upper limits of this structure of exercise prices, $\triangle K_n$ is calculated as the absolute value of the difference between the exercise price at the limit and its adjacent price. It is clear that this marginal contribution increases with the option premium irrespective of option type. It also appears from Tables 3A and 3B that increasing exercise prices tend to drive the marginal contribution down (up) for call (put) options.

The cumulative contributions of option prices $\sum_n (\triangle K_n / K_n^2) e^{r\tau_m} Q(K_n)$ are subsequently used to estimate the implied volatility from near-term and next-term maturities. The implied variance can be estimated on the basis of at-the-money exercise price and forward index level as follows.

$$\sigma_m^2 = \frac{2}{\tau_m} \sum_n \frac{\triangle K_n}{K_n^2} e^{r\tau_m} Q(K_n) - \frac{1}{\tau_m}\left(\frac{F_m}{K_0} - 1\right)^2 \tag{5}$$

In the above example, the implied variance is estimated at $\sigma_1^2 = 0.067667454$ with $\sum_n \frac{\triangle K_n}{K_n^2} e^{r\tau_1} Q(K_n) = 0.000905085$ for near-term maturity and $\sigma_2^2 = 0.048231978$ with $\sum_n \frac{\triangle K_n}{K_n^2} e^{r\tau_2} Q(K_n) = 0.002963884$ for next-term maturity. The interpolation of these estimates is made in order to arrive at a single measure of implied variance for the hypothetical option with 30 days to expiration. The estimate of implied volatility on February 27, 2001 is equal to $\sqrt{\sigma^2} = 0.00508908^{1/2} = 22.56\%$ based on the following measure of implied variance

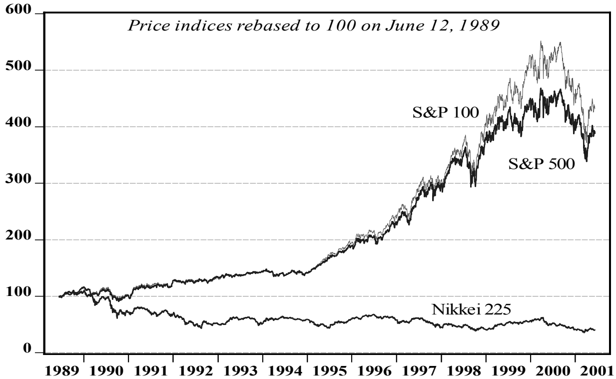
$$\sigma^2 = \left(\tau_1 \sigma_1^2 \left[\frac{N_{\tau 2} - N_{30}}{N_{\tau_2} - N_{\tau_1}}\right] + \tau_2 \sigma_2^2 \left[\frac{N_{30} - N_{\tau_1}}{N_{\tau_2} - N_{\tau_1}}\right]\right) \times \frac{N_{365}}{N_{30}} \tag{6}$$

where N_{t_1} is the number of minutes to near-term expiration (14030), N_{τ_2} is the number of minutes to next-term expiration (64430), N_{30} is the number of minutes to the 30-day expiration of the hypothetical option (43200) and N_{365} is the number of minutes per year (525600).

3. Distributional Properties of Implied Volatility Indices

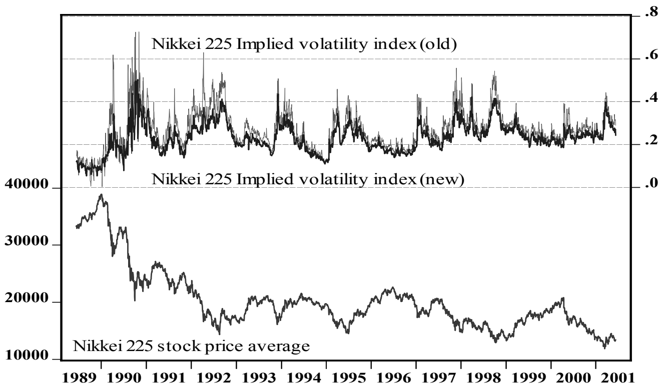
The closing levels of S&P 100 and the model-free new VIX based on S&P 500 index are obtained from Thomson Financial Datastream database. Similar benchmarks are estimated for the Nikkei 225 stock index options. Such estimates of implied volatility index provide forward-looking estimates of short-term volatility which can be compared across markets and over time. It appears from Figure 1 that over the sample period, the S&P 100 and S&P 500 benchmarks reflected significant increases in stock prices, reaching five-fold levels toward the end of the sample period. This is in sharp contrast with the behavior of prices in the Japanese stock market where the Nikkei 225 stock average exhibited a monotonous decrease following the burst of the asset bubble in the early 1990s.

Figure 1. Time-series of stock price indices



In the presence of these two opposing trends of stock price movements, it is interesting to examine the behavior of volatility expectations across markets.

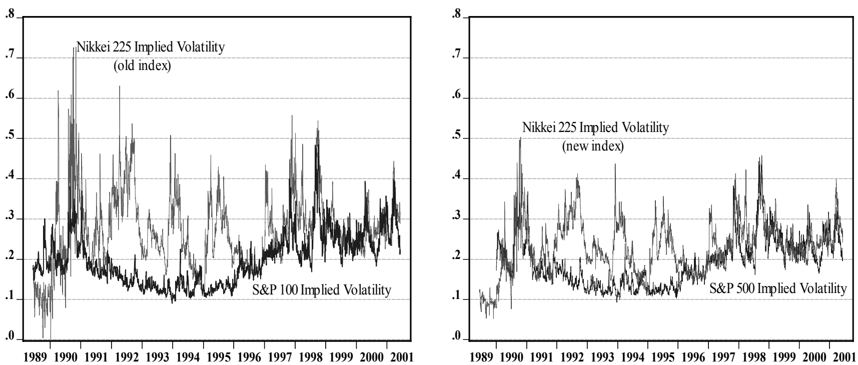
Figure 2. The behavior of the Japanese stock prices and implied volatility indices



Judging from Figure 2, implied volatility in the Japanese market appears to follow cyclical patterns with several peaks and troughs seemingly related to the behavior of the underlying spot prices. An increase in implied volatility is reflective of expectations of rising stock market volatility. This can in turn, provide some useful signals on the future tendency of stock prices to increase or decrease in response to significant economic events. It seems that the various patterns in volatility expectations are well described despite methodological discrepancies between the original and new implied volatility indices.

There appear to be clear discrepancies in the behavior of benchmarks of implied volatility across the Japanese and US markets. Indeed, as Figure 3 indicates, the original implied volatility index for the Japanese market reveals large spreads in comparison with the S&P 100 index. The differences between these benchmarks are more salient over the 1991～1995 period. These gaps remain with respect to the new Nikkei 225 implied volatility index and the new VIX benchmark based on the S&P 500 index. There is evidence that the spreads are relatively lower in magnitude. In fact, the levels of the new implied volatility indices seem to be on average, lower than the original levels, an observation

Figure 3. Implied volatility in the Japanese, and US stock markets



which may be attributed to differences in the estimation methodologies. The model-free approach is less likely to yield extreme values of implied volatility.

The distributional properties of implied volatility are described in the statistics reported by Table 4. The average implied volatility is indeed lower for the new index relative to the original one, irrespective of markets. It is also important to note that market volatility is expected to be on average higher for the Japanese market compared to the US market. This may be related to the persistent trend for negative returns on Japanese stocks. Also, volatility expectations cannot be assumed to be constant. They vary over time and across markets. Thus, the degree of variability of the implied volatility index, measured by its standard deviation, is also of importance as it provides some indication on the fluctuations of volatility expectations. The standard deviation of implied volatility indices is found to be highest for the original Nikkei 225 implied volatility index. It also appears to be higher for the Japanese market than the US market. The higher fluctuations of volatility expectations in the Japanese market cannot thus be merely attributed to differences in the methodological approaches. The higher variability of volatility expectations is in line with the higher fluctuations of

Table 4. Distributional Moments of stock market returns and implied volatility

Time series	Mean	Median	Std. Dev.	Skewness	Kurtosis	J-Bera	ADF
	Stock market returns						
Nikkei 225 index	-0.00029	0.00000	0.01451	0.29313	7.31525	2472.568	-42.092[c]
S&P 100 index	0.00047	0.00019	0.01011	-0.28705	8.00760	3312.256	-57.307[b]
S&P 500 index	0.00043	0.00019	0.00956	-0.34646	8.16625	3531.948	-55.697[b]
	Implied volatility index						
Nikkei 225 index (old)	0.27142	0.26280	0.09112	0.60552	4.26489	399.801	-5.047[b]
Nikkei 225 index (new)	0.23064	0.22840	0.06791	0.28008	3.43134	65.188	-5.328[b]
S&P 100 index	0.19770	0.18685	0.06377	0.80916	3.67709	401.344	-5.267[a]
S&P 500 index	0.19139	0.18290	0.05945	0.85346	3.78584	439.037	-4.010[a]

Notes: The sample period of daily observations extends from June 12, 1989 to June 8, 2001. The time series of benchmark implied volatility starts on January 2, 1990 for the S&P 500. ADF statistics refer to the augmented Dickey-Fuller results for unit root tests. The appropriate lag order is determined according to Schwarz information criterion and additional lags are included to eliminate ARCH effects in the residuals. The superscripts [a], [b] and [c] refer to tests of stationarity in the levels of return and implied volatility series with intercept and trend terms, with intercept only, and with neither intercept nor trend terms, respectively. The 1% critical values for these cases are -3.961, -3.432 and -2.566, respectively.

Japanese stock market returns. The higher distributional moments suggest leptokurtic distributions of stock market returns as well as implied volatilities. Judging from the Jarque-Bera statistics, there is little evidence that returns, as well as implied volatilities, follow normal distributions. However, based on the Augmented Dickey-Fuller tests, these time-series are found to be stationary.

4. The Dynamics of implied volatility and its relationship with stock market returns

Given the observed patterns in implied volatility and its distributional properties, it is possible to examine more formally the dynamics of volatility expectations and the relationship with stock market returns. This analysis can shed light on the usefulness of implied volatility indices in providing information on the changing degree of investors' fear or exuberance over time. This examination is based on the following autoregressive model where past observations of returns are included in the set of conditioning variables.

$$v_t = w_0 + \sum{}_k \gamma_k v_{t-k} + \lambda r_{t-1} + \theta v^{\bullet}_{t-1} \qquad (7)$$

The regression model described by equation (7) describes volatility expectations as function of their past levels and the sign and magnitude of past returns. Depending on the sign and significance of the autoregressive γ coefficients, it is possible to determine whether implied volatility is associated with a long memory process with positive γ parameters, in the sense that rising volatility expectations are followed by incremental increases in implied volatility or alternatively governed by mean reversion process with negative γ parameters. The test of significance of the relationship of implied volatility with past returns can shed light on the impact of shocks to the return-generating process on volatility expectations in options markets. For a given implied volatility index, the independent variable $v^{\bullet}$ denotes the corresponding volatility expectations in the alternative market. For instance, it refers to the S&P 500 volatility expectations in the case of estimating the dynamics of the new Nikkei 225 implied volatility index.

Table 5. Determinants of the level of implied volatility indices

Model Parameters	Nikkei 225 (old)	Nikkei 225 (new)	S&P 100	S&P 500
w_0	0.0045**	0.0040*	0.0036*	0.0033*
γ_1	0.5586*	0.8210*	0.8566*	0.9133*
γ_2	0.2358*	0.0780*	0.0385	-0.0447
γ_3	0.0722*	-0.0120	-0.0250	0.0241
γ_4	0.0805*	0.0506*	0.0676*	0.0330
γ_5	-0.0561*	0.0248	-0.0016	0.0484***
γ_6	-0.0257	-	-0.0017	-0.0463***
γ_7	0.0310	-	-0.0238	-0.0026
γ_8	0.0153	-	0.0762*	0.0200
γ_9	-0.0639*	-	-	0.0430**
γ_{10}	0.1110*	-	-	-
λ	-0.3102*	-0.1156*	0.0022	-0.0379
θ	0.0337*	0.0257*	-0.0035	-0.0044
LB(1)	0.923	0.965	0.915	0.886
LB(10)	0.999	0.091	0.421	0.331
LogLikelihood	6347.93	8074.83	8997.49	8982.48

Notes: The sample period of daily observations extends from June 12, 1989 to June 8, 2001. The time series of benchmark implied volatility starts on January 2, 1990 for the S&P 500. Significance at the 1%, 5% and 10% levels are denoted by *, ** and ***, respectively. The autoregressive model (7) is estimated with respect to the levels of implied volatility. LB(q) denotes the probability associated with the Ljung-Box test for serial correlation in the residuals up to the q^{th} order.

It is clear from the model estimates reported in Table 5 that there is a tendency for implied volatility to depend significantly on its historical levels. Indeed, irrespective of stock markets and of calculation methods, all implied volatility indices are associated with positive and significant first autoregressive γ_1 coefficients, which suggest that surges in volatility expectations are likely to be followed by further increases. However, judging from the statistical insignificance of subsequent autoregressive γ coefficients, the memory process of implied volatility in the US market is not as significant as the process governing volatility expectations in the Japanese market. The level of implied volatility is found to depend on past observations of stock market returns in the Japanese market, but not in the US market. The evidence suggests that implied volatility indices are likely to increase following negative returns. Implied volatility in the Japanese market is also found to be sensitive to past levels of volatility expectations in the US market, judging from the insignificance of θ coefficients for the Japanese series. However, the levels of implied volatility in US market are found to be generated independently of volatility expectations in alternative markets.

Table 6. The dynamics of implied volatility indices

Model Parameters	Nikkei 225 (old)	Nikkei 225 (new)	S&P 100	S&P 500
w_0	-0.0001	0.0045	0.0032	0.0001
γ_1	-0.4264*	-0.1621*	-0.1337*	-0.0796*
γ_2	-0.1784*	-0.0768*	-0.0970*	-0.1249*
γ_3	-0.1054*	-0.0927*	-0.1248*	-0.1010*
γ_4	-0.0260	-0.0454**	-0.0564*	-0.0685*
γ_5	-0.0868*	-0.0132	-0.0567*	-0.0197
γ_6	-0.1025*	-0.0609*	-0.0578*	-0.0655*
γ_7	-0.0731*	-0.0626*	-0.0809*	-0.0676*
γ_8	-0.0510*			-0.0471**
γ_9	-0.1189*			
λ	-0.2867*	-0.1117*	0.0089	-0.0346
ϑ	0.4600*	0.1486*	-0.0067	-0.0092
LB(1)	0.366	0.547	0.880	0.864
LB(10)	0.840	0.492	0.416	0.409
LogLikelihood	6392.42	8072.36	8986.81	8975.35

Notes: The sample period of daily observations extends from June 12, 1989 to June 8, 2001. The time series of benchmark implied volatility starts on January 2, 1990 for the S&P 500. Significance at the 1%, 5% and 10% levels are denoted by *, ** and ***, respectively. The autoregressive model (7) is estimated with respect to first differences in implied volatilities on both sides of the equation. The estimates of the drift w_0 are expressed in percent terms. LB(q) denotes the probability associated with the Ljung-Box test for serial correlation in the residuals up to the q^{th} order.

The dynamics of volatility expectations can be examined according to model (7) for the series of first differences rather than the levels of implied volatility. The results reported by Table 6 indicate that variations in implied volatility are significantly affected by historical trends. The negative sign associated with all autoregressive γ coefficients suggests however that these dynamics are driven by mean reversion. Positive changes in implied volatility are likely to be followed by decreases over several trading days. There is also evidence that changes in implied volatility in the Japanese market are negatively related to returns and positively correlated with the first differences in US implied volatility. The negative sign associated with λ coefficients for Japanese stock market returns suggests that implied volatility is likely to increase in bearish markets and decrease in bullish markets. Again, this is not true of the US market where implied volatility is found to be insensitive to market returns and past changes in volatility expectations in the Japanese market.

5. Conclusion

This paper introduced two benchmarks of implied volatility for the Japanese stock market, which are directly comparable to the original S&P 100 and new S&P 500 implied volatility indices. The calculation methodologies were reviewed in order to distinguish the differences between the Black-Scholes model-based approach in estimating the original index and the model-free approach underlying the new volatility index. There is no index of implied volatility available for the Japanese market despite the importance of the Nikkei 225 index, which constitutes the underlying asset of derivatives trading in the Japanese, Singapore and US markets. These benchmarks can be useful in providing some measure of volatility expectations, which is not based on historical data from the spot market but on investors' beliefs from the derivatives market. They are also useful for comparative purposes by relating volatility expectations across international markets. The examination of the usefulness of implied volatility for forecasting purposes falls beyond the scope of this paper. However, it is shown that implied volatility is likely to follow cyclical patterns which may be related to shifts in the return-generating process.

The time-series of implied volatility also indicate that wider spreads between the Japanese and US volatility expectations in the early 1990s are no longer observed in more recent years. The lower significance of the spread between implied volatilities across countries may be to some extent, attributed to more accurate measures of volatility expectations and/or the stronger integration of international stock markets. As far as the Japanese market is concerned, the empirical evidence suggests that implied volatility is likely to increase in bearish markets and decrease in bullish markets. It is also found to be sensitive to the implied volatility in US markets. These results do not apply for volatility expectations in the US markets where implied volatility seems to behave independently of past observations of returns and/or implied volatility in alternative markets.

Further examination of the dynamics of implied volatility is warranted given the potential benefits such an analysis holds for the purposes of forecasting short-term volatility, risk-hedging and monetary policy-making. It is important for policy-makers including central bankers for instance, to gain useful information on market participants' expectations about future volatility and its relationship with the release of economic reports and anticipations of shifts in monetary policy, among others. Together with Value-at-Risk analysis using implied volatility indices, these important issues constitute interesting avenues for future research.

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